

§ 10.2 毕奥—萨伐尔定律

一. 毕奥—萨伐尔定律

静电场：取 $dq \longrightarrow d\vec{E} \longrightarrow \vec{E} = \int d\vec{E}$

磁 场：取 $Id\vec{l} \xrightarrow{\text{?}} d\vec{B} \longrightarrow \vec{B} = \int d\vec{B}$

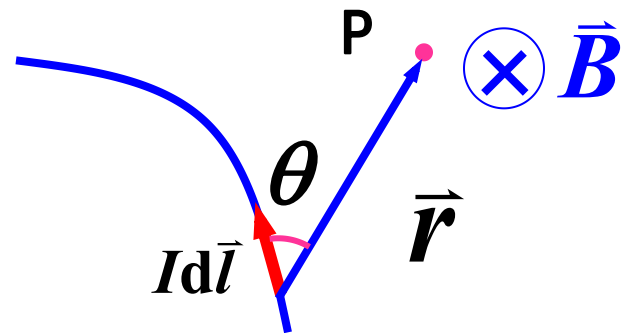
毕—萨定律：
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \vec{r}}{r^3}$$
 $\vec{r}$ — 电流元到待求点的位矢

$$\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$

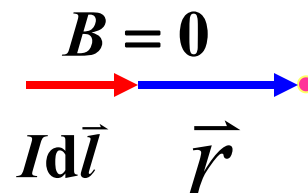
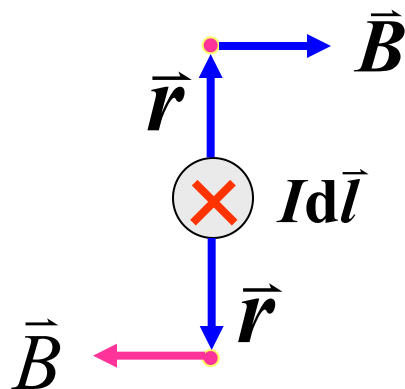
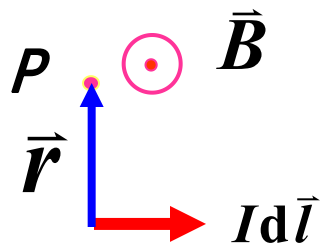
真空中的磁导率

大小：
$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

方向：垂直电流元与位矢组成平面；右螺旋法则



例如：



二. 毕—萨定律的应用

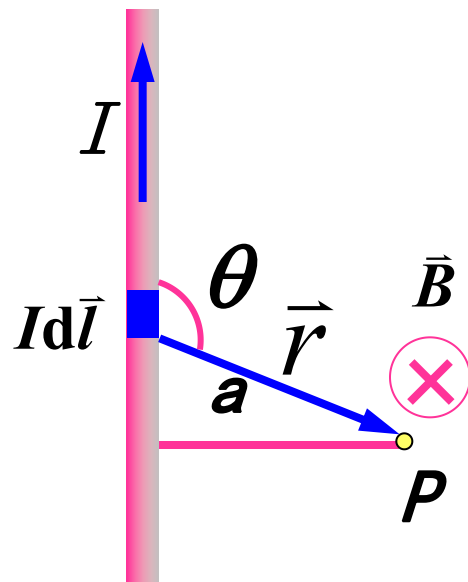
1. 载流直导线的磁场

求距离载流直导线为 a 处
一点 P 的磁感应强度 $\vec{B}$

解

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

$$B = \int dB = \int \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$



根据几何关系

$$B = \int dB = \int \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$$

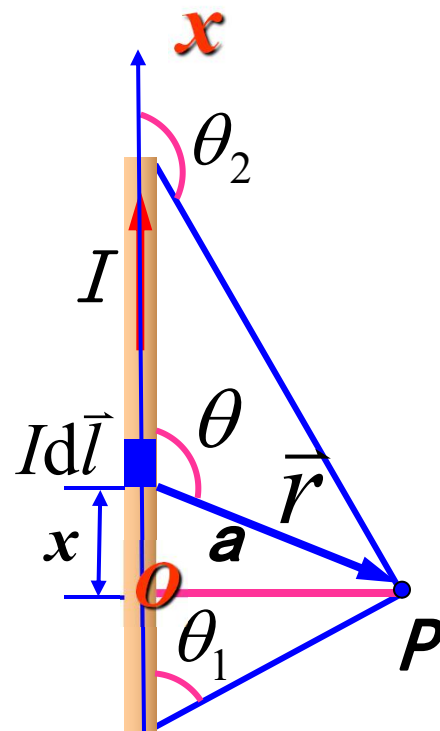
$$r = a \csc \theta$$

$$x = a \cot(\pi - \theta) = -a \cot \theta$$

$$dl = dx = a \csc^2 \theta d\theta$$

$$B = \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin \theta d\theta$$

$$= \frac{\mu_0 I}{4\pi a} (\cos \theta_1 - \cos \theta_2)$$



★ 讨论

$$B = \frac{\mu_0 I}{4\pi a} (\cos \theta_1 - \cos \theta_2)$$

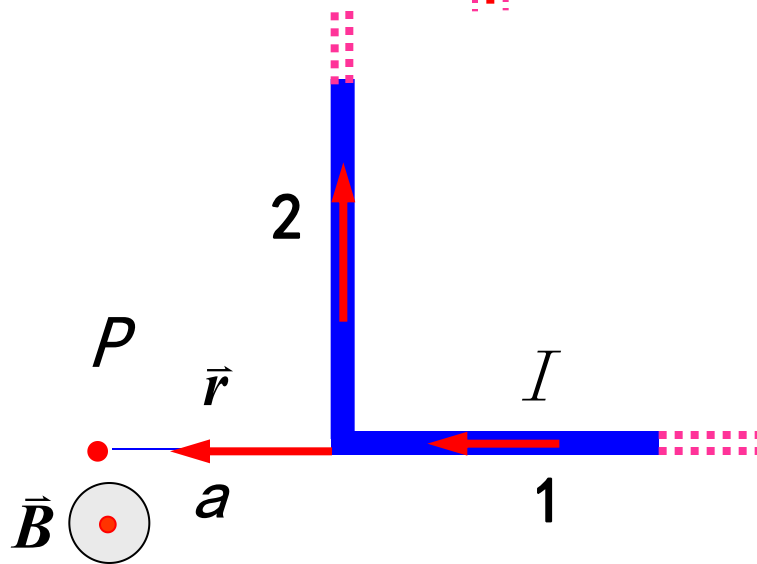
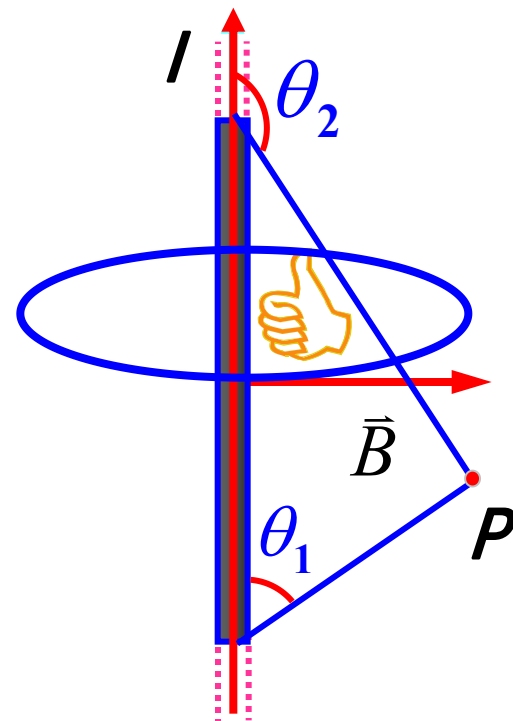
(1) 无限长直导线 $\theta_1 \rightarrow 0$ $\theta_2 \rightarrow \pi$

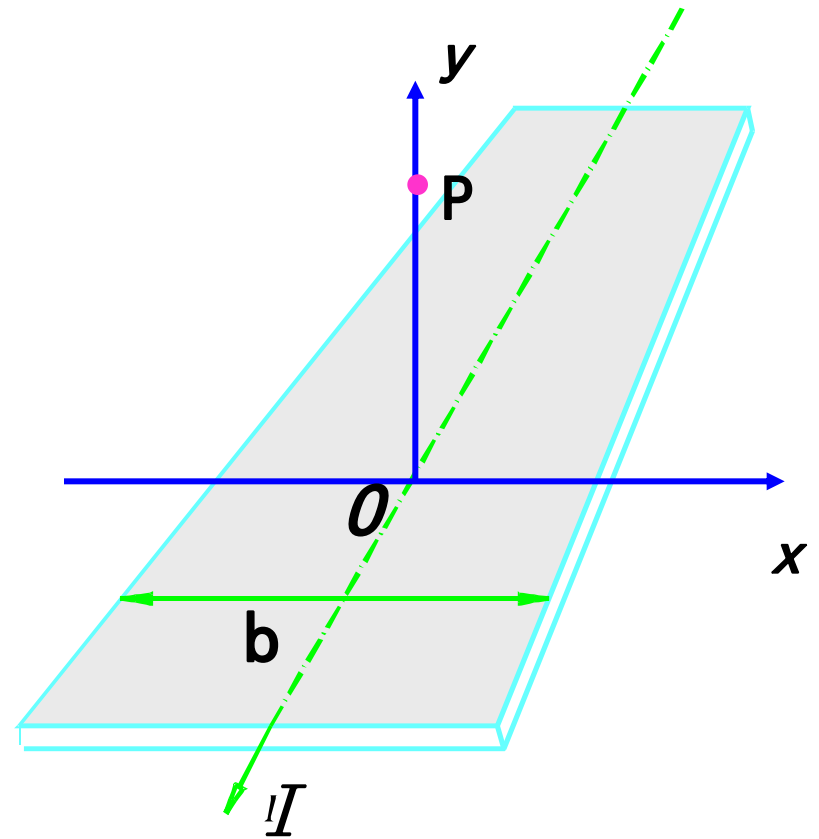
$$B = \frac{\mu_0 I}{2\pi a} \quad \text{方向：右螺旋法则}$$

(2) 任意形状直导线

$$B_1 = 0$$

$$\begin{aligned} B_2 &= \frac{\mu_0 I}{4\pi a} (\cos 90^\circ - \cos 180^\circ) \\ &= \frac{\mu_0 I}{4\pi a} \end{aligned}$$





$$B_p = \frac{\mu_0 I}{\pi b} \arctan \frac{b}{2y}$$

2. 载流圆线圈的磁场

求轴线上一点 P 的磁感应强度

$$dB = \frac{\mu_0}{4\pi} \frac{Idl}{r^2} = \frac{\mu_0}{4\pi} \frac{Idl}{(R^2 + x^2)}$$

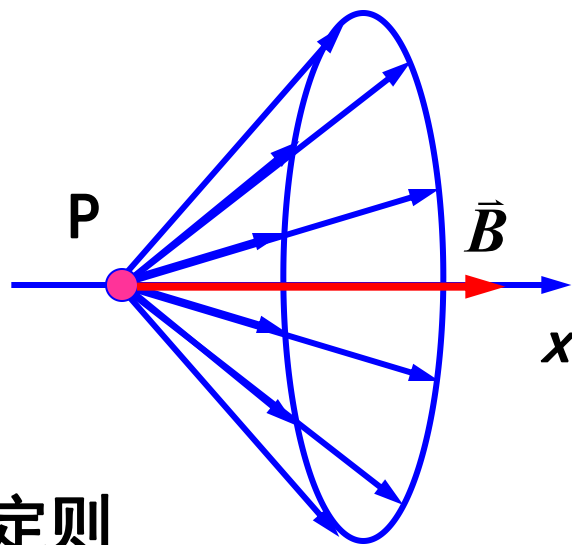
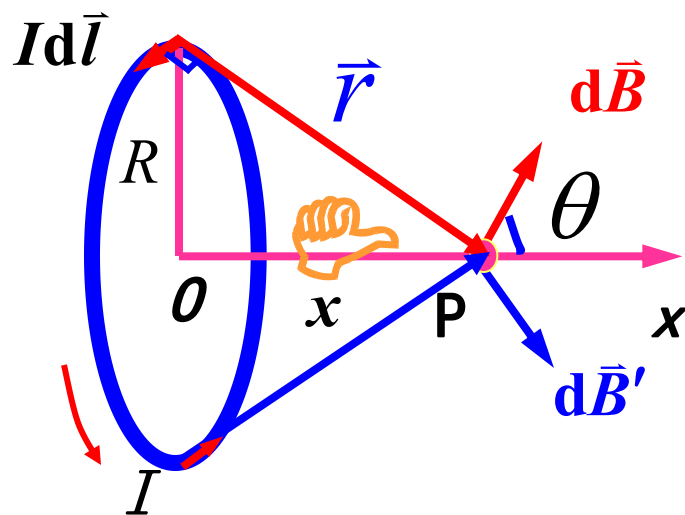
根据对称性 $B_{\perp} = 0$

$$B = \int dB_x = \int dB \cos \theta = \int \frac{\mu_0}{4\pi} \frac{Idl}{r^2} \cos \theta$$

$$\cos \theta = \frac{R}{r} = \frac{R}{(R^2 + x^2)^{1/2}}$$

$$B = \frac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}$$

方向满足右手定则



讨论

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

(1) $x = 0$ 载流圆线圈的圆心处

$$B = \frac{\mu_0 I}{2R}$$

如果由 N 匝圆线圈组成 $B = \frac{\mu_0 N I}{2R}$

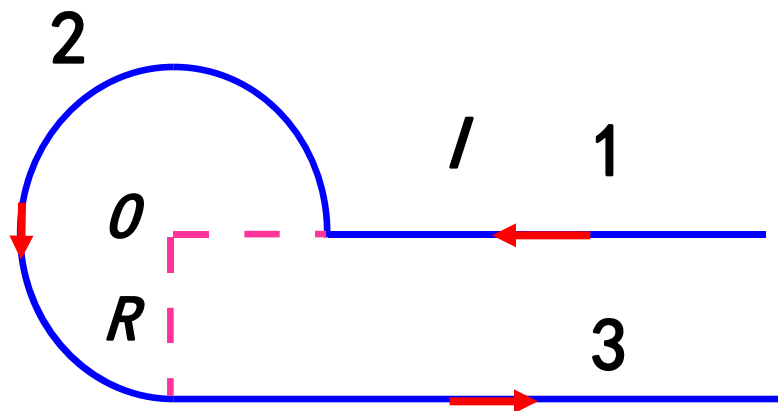
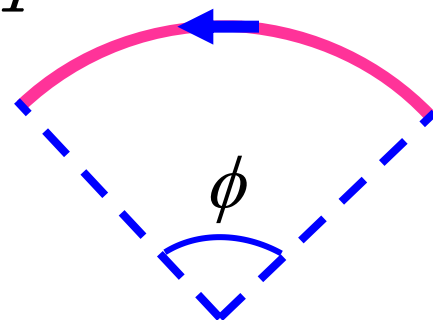
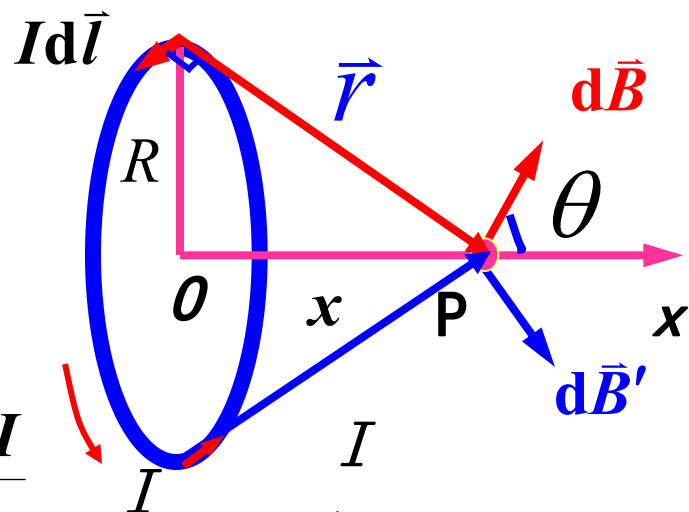
(2) 一段圆弧在圆心处产生的磁场

$$B = \frac{\mu_0 I}{2R} \cdot \frac{\phi}{2\pi} = \frac{\mu_0 I \phi}{4\pi R}$$

例如 右图中, 求 O 点的磁感应强度

解 $B_1 = 0$

$$B_2 = \frac{\mu_0 I}{4\pi R} \cdot \frac{3\pi}{2} = \frac{3\mu_0 I}{8R}$$

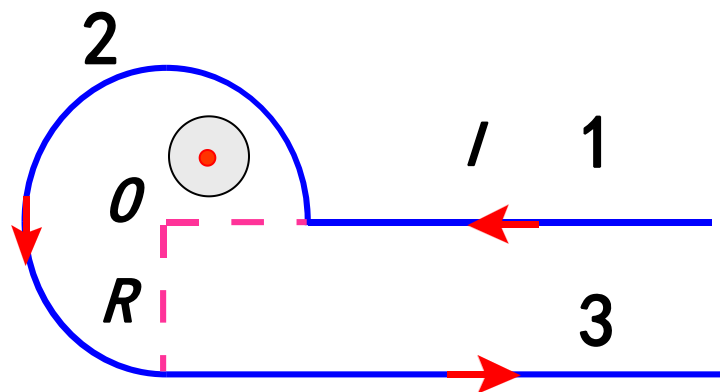


$$B_3 = \frac{\mu_0 I}{4\pi R} (\cos \theta_1 - \cos \theta_2)$$

$$= \frac{\mu_0 I}{4\pi R}$$

$$\theta_1 = \pi/2$$

$$\theta_2 = \pi$$



$$B = B_1 + B_2 + B_3$$

(3) $x \gg R$ ➡ $B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$

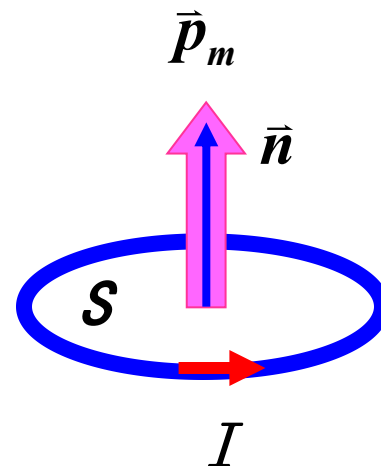
$$B \approx \frac{\mu_0 I R^2}{2x^3} \cdot \frac{\pi}{\pi} = \frac{\mu_0 I S}{2\pi x^3}$$

定义

$$\vec{p}_m = IS\vec{n}$$

磁矩

$$\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{p}_m}{x^3}$$

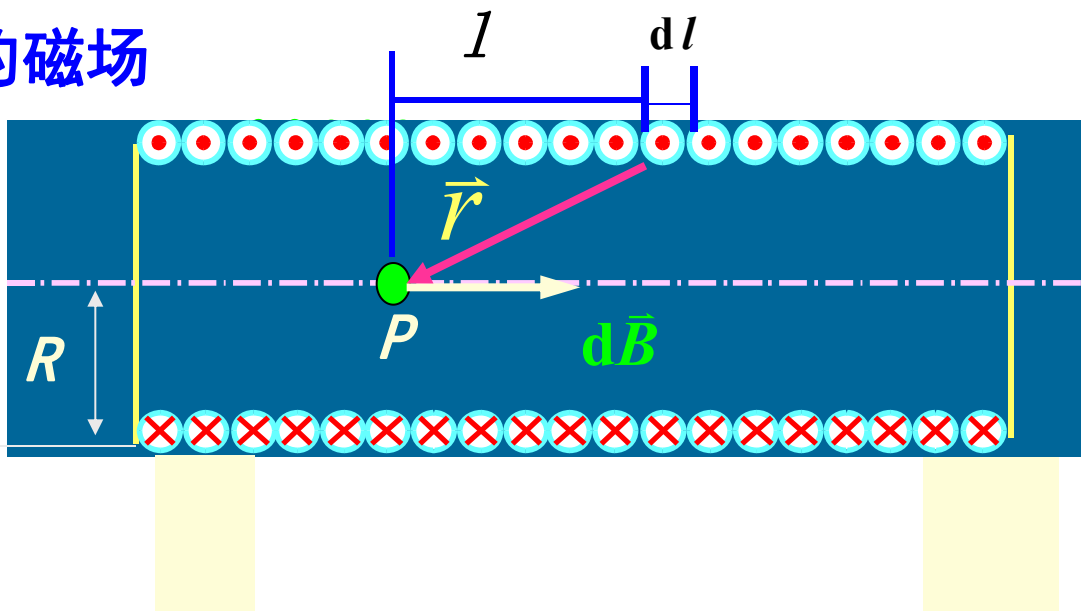


3. 载流螺线管轴线上的磁场

已知螺线管半径为 R

单位长度上有 n 匝

$$dI' = In dl$$



$$dB = \frac{\mu_0 R^2 dI'}{2(R^2 + l^2)^{3/2}} = \frac{\mu_0 R^2 I n dl}{2(R^2 + l^2)^{3/2}}$$

$$R^2 + l^2 = R^2 \csc^2 \beta$$

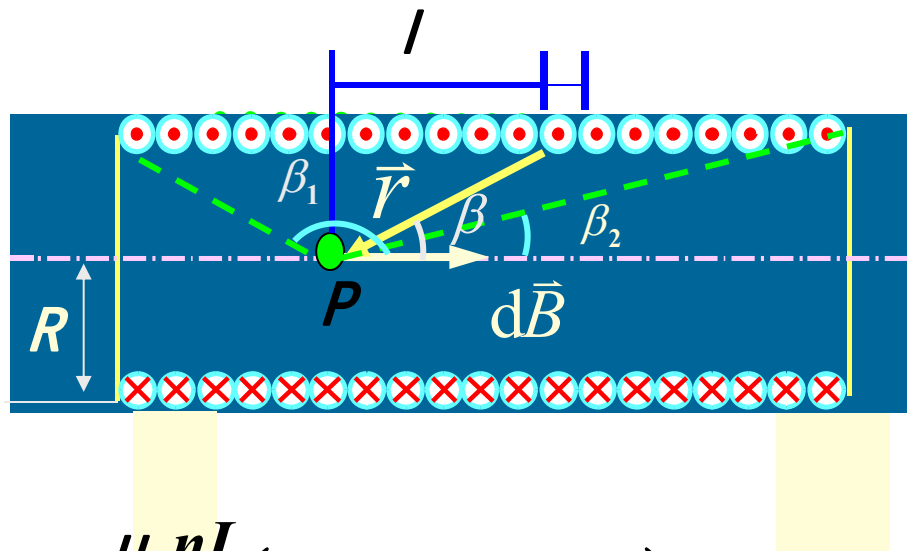
$$l = R \cot \beta$$

$$dl = -R \csc^2 \beta d\beta$$

$$dB = -\frac{\mu_0}{2} nI \sin \beta d\beta$$

$$B = \int_{\beta_1}^{\beta_2} -\frac{\mu_0}{2} nI \sin \beta d\beta$$

$$= \frac{\mu_0 nI}{2} (\cos \beta_2 - \cos \beta_1)$$



讨论

(1) 无限长载流螺线管 $\beta_1 \rightarrow \pi, \beta_2 \rightarrow 0 \rightarrow B = \mu_0 nI$

(2) 半无限长载流螺线管 $\beta_1 \rightarrow \pi/2, \beta_2 \rightarrow 0 \rightarrow B = \mu_0 n \frac{I}{2}$